Abraham Robinson
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Abraham Robinson was born in Waldenburg, Germany (now Walbrzych, Poland) on October 6, 1918. His father, who died before Abby's birth, was a man of high intellectual attainments and served as private secretary and biographer to Theodore Herzl's successor as President of the World Zionist Organization, David Wolfson. His mother, daughter of a well-known Rabbi, was a woman of wide culture. With the advent of Hitler, the mother emigrated to Israel with Abby and his older brother Saul. The two brothers grew up in Jerusalem, where they were held up as an example of adversity overcome by hard work, discipline, breeding, and great natural gifts. Abby excelled in his studies, and when he was 18 entered The Hebrew University to study mathematics. He became an enthusiastic participant in the mathematical seminars and his remarkable progress led Professor Frankel to announce publicly in 1938 that there was a second-year student at the University to whom he had nothing to teach adding "he knows as much mathematics as an advanced mathematician".

The years 1938–39 were difficult years. Apart from working to support himself, he and his brother cared for their mother whose health was poor. Moreover, those were years of armed conflict between the Jewish and Arab communities. Since 1936, Palestine had been in the grips of major Arab uprisings directed against the Jews. Attacks were being mounted against Jewish settlements all over Palestine. Abby was one of the students selected by the illegal Jewish defense organization, the Haganah, for junior officer training, taking his training at Kibbutz Kiryat Anavim. As an active member of the Haganah, he spent many nights on guard duty and other missions. In 1939, though he had not yet formally completed his undergraduate studies, he was awarded a fellowship to do postgraduate research at the Sorbonne. When France fell, escape was imperative and he and Jacob Talmon, a history student on a similar fellowship, made their way, largely on foot, to Bordeaux where they boarded one of the last refugee boats for England. They spent about 10 days in a refugee camp outside London but reestablished contact with Shimon Abramsky, now Professor of Jewish History at London and Oxford Universities, a fellow trainee at Kiryat Anavim who had been stranded in England at the outbreak of war, and with his help they were released. They spent many evenings conversing far into the night on problems of mutual interest, particularly philosophy, the question of peace, and the eternal peace as formulated by Kant. Abramsky, a philosopher, was deeply impressed by Abby's erudition not only in philosophy, but the whole gamut of German culture and especially Goethe. Abby was even more at home

in medieval and modern Hebrew culture, quoting extensively from Bialik, and displaying remarkable familiarity with Maimonides.

Later, Abby succeeded in joining the Free French. When his gifts became known to the British, they arranged for a transfer to the Royal Aircraft Establishment in Farnborough as a scientific officer.

It soon became apparent that Robinson was an exceptionally good applied mathematician. He had an outstanding ability for understanding physics, so that his applications of mathematics were extraordinarily stimulating as well as elegant. In a very short time he mastered wing theory and in a series of influential papers, expanded its frontiers particularly in the field of supersonic aerodynamics.

After the war, the College of Aeronautics was founded at Cranfield and Abby was appointed a Senior Lecturer. At that time he was also awarded a M. Sc. degree from The Hebrew University. He produced a number of important papers at Cranfield establishing him as a leading expert on delta wing theory. Together with a research student J. A. Laurmann, he undertook the writing of a major book and their *Wing Theory* published in 1956 became a standard international text. Simultaneously, he developed his interest in Logic and every week went down to Berkly College to see his supervisor, inasmuch as he had registered for a London University Ph.D. He was awarded a Ph.D. in 1949.

He was appointed Deputy Head of the Department of Aerodynamics at Cranfield. He left that position to accept a position in applied mathematics, at the University of Toronto, Canada, where he stayed from 1951 to 1957. He also received a D.Sc. from London University in 1957. Thereafter he was professor and later chairman at The Hebrew University, Jerusalem, Israel (1957–1962), Professor of Mathematics and Philosophy at the University of California, Los Angeles (1962–1967), and Professor of Mathematics at Yale University since 1967 (Sterling Professor of Mathematics since 1971).

He was a Visiting Professor at the Universities of Princeton, Paris, Rome, Tubingen, Heidelberg, at the California Institute of Technology, and at the Weizmann Institute, Rehovoth, and a Visiting Fellow at St. Catherine's College, Oxford. His activities also included membership on the Fluid Motion Committee of the Aeronautical Research Council of Great Britain. In 1972 he was elected a fellow of the American Academy of Arts and Sciences and in 1973 he received the Brouwer Medal from the Dutch Mathematical Society. In April 1974 he was elected posthumously as member in the National Academy of Sciences. He was President of the Association for Symbolic Logic from 1968 to 1970, and had been a consultant to The National Science Foundation and to the I.B.M. Corporation. He published ten books and 135 papers in pure and applied mathematics, in philosophy, and economics.

Robinson worked in widely separated areas of science. However, the common denominator to much of his research was his interest in applications. He had always been fascinated by the problem of fashioning or refashioning a formal framework in order to fit a given problem, whether in physics or in pure

mathematics. Within classical applied mathematics, he had been concerned chiefly with fluid mechanics, more particularly with the determination of the pressures and forces that act on a body in flight, under steady or unsteady conditions, from subsonic to supersonic speeds. Some of his better known contributions in this area were concerned with delta wings and related shapes, while other papers dealt with the motion of small bodies in a viscous fluid and with the propagation of disturbances in fluids and solids. One of these led to an early example of a precise theory for a mixed boundary value problem for hyperbolic differential equations.

However, Robinson's major effort went into the study of the relations between logic and mathematics proper. In his Ph.D. dissertation "On the Metamathematics of Algebraic Systems" (1949) he helped to lay the foundations of the branch of logic now known as Model Theory. He discussed generally, and in special cases, the mutual relationship between sets of axioms and the classes of structures (models) which satisfy them.

In 1954 Robinson produced a widely applicable test (the model completeness test) for proving the completeness of various algebraic theories. Beginning in 1969, Robinson was able to establish that even in arithmetic one can introduce structures which are analogous to algebraically closed fields, and that the theory of these structures is complete (or, which is the same, such that any two of these structures are elementarily equivalent).

Robinson's most striking innovation was Nonstandard Analysis, which he founded. This theory, which was introduced by him from 1960 on, makes use of model theoretic notions and contributions in order to provide for the first time a satisfactory solution to Leibniz's problem of developing the differential and integral calculus by means of infinitesimals. It turned out that the ideas which led to Nonstandard Analysis can be generalized so as to apply also to topological spaces and many other areas of mathematics. The method has been used successfully for the solution of problems in functional analysis and in complex variable theory and, more recently, in physics and mathematical economics. His last paper, a joint work with Professor Peter Roquette of Heidelberg, is a remarkably significant application of nonstandard analysis to number theory.

Robinson was distinguished not only as a thinker, but also as a human being. There was scarcely a human activity in which he was not interested, and his views were always characterized by deep insight, passion, and wisdom. Even during the London blitzes and the darkest moments of Nazi brutality he offered constant encouragement to his friends Abramsky and Talmon, and he expressed immense hope in the future.

In his life he was guided by an almost paradoxical principle of complementarity: on the one hand, deliberately noncommittal about transcendental descriptions, on the other hand, wholeheartedly committed to prescriptive conduct. These complementary principles characterized Abby's approach to research as well as to life. In his address to the 1964 International Congress for Logic, Methodology, and Philosophy of Science held in Jerusalem, Abby stated:

My position concerning the foundations of Mathematics is based on the following two main (points or) principles:

i) Infinite totalities do not exist in any sense of the word (i.e. either really or ideally). More precisely, any mention or purported mention of infinite totalities is, literally, *meaningless.*

ii) Nevertheless, we should continue the business of Mathematics "as usual", i.e. we should act *as if* infinite totalities really existed.

These complementary principles: rejection of transcendental descriptions of existence and acceptance of prescriptions for moral conduct, underlay his personal philosophy.

The commitment to moral conduct was evident throughout Abby's life. He was a man of peace, sensitive to the rights of the underprivileged, always ready to do more than his share in any joint effort. Even during his last painful months, he was loath to ask for special privileges.

On the other hand, when Professor Kurt Godel, whom Abby considered the greatest logician of our time, wrote of Abby as he lay dying "that man is not just a collection of molecules". Abby's reaction was "In such matters my philosophy is deliberately incomplete". Robinson died on April 11, 1974 after an illness of six months.

Professor Godel wrote his widow "So, please, let me say now how deeply I regret the premature death of your husband, whom I valued very highly indeed, not only as a personal friend, but also as the one mathematical logician who accomplished incomparably more than anybody else in making mathematical logic fruitful for mathematics. I am sure his name will be remembered by mathematicians for centuries."

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BIBLIOGRAPHY OF ABRAHAM ROBINSON

BOOKS

On the Metamathematics of Algebra, North-Holland Publ. Co., Amsterdam, 1951, ix + 195 pp.

Théorie Métamathématique des Ideaux, Gauthier-Villars, Paris, 1955, ix + 186 pp.

Wing Theory (with J. A. Laurmann), Cambridge Univ. Press, Cambridge, England, 1956, ix + 569 pp.

Complete Theories, North-Holland Publ. Co., Amsterdam, 1956, ix + 129 pp.

Introduction to Model Theory and the Metamathematics of Algebra, North-Holland Publ. Co., Amsterdam, 1963, ix + 284 pp. Translated into Russian 1967, into Italian 1974, new edition 1974.

Numbers and Ideals, Holden-Day, San Francisco, 1965, ix + 106 pp.

Non-Standard Analysis, North-Holland Publ. Co., Amsterdam, 1966, new edition 1974, ix + 293 pp.

Contributions to Non-Standard Analysis (co-editor with W. A. J. Luxemburg), North-Holland Publ. Co., Amsterdam, 1972, ix + 289 pp.

Nonarchimedean Fields and Asymptotic Expansions (with A. H. Lightstone), North-Holland Publ. Co., Amsterdam, 1975, ix + 204 pp.

Algebra — A Model Theoretic Viewpoint (with V. B. Weispfenning), Springer-Verlag, to appear.

PAPERS

On the independence of the axioms of definiteness, J. Symbolic Logic **4** (1939), 69–72.

On nil-ideals in general rings. Galley proofs recovered posthumously. To appear in Selected Papers of Abraham Robinson, Yale University Press, New Haven, Connecticut.

On a certain variation of the distributive law for a commutative algebraic field, Proc. Roy. Soc. Edinburgh, Sect. A, **61** (1941), 93–101. MR 3 # 101.

A note on the interpretation of $V - g$ records (with P. E. Montagnon and S. V. Fagg), Ministry of Supply (London), Aeronaut. Res. Council, Rep. and Memoranda No. 2097 (9460), (1945), 8 pp.

The aerodynamic loading of wings with endplates, Ministry of Supply (London), Aeronaut. Res. Council, Rep. and Memoranda No. 2342 (8615), 1945 (1950), 13 pp.

The wave drag of diamond-shaped aerofoils at zero incidence, Ministry of Supply (London), Aeronaut. Res. Council, Rep. and Memoranda No. 2394 (9780), 1946 (1950), 6 pp.

Shock transmission in beams, Ministry of Supply (London), Aeronaut. Res. Council, Rep. and Memoranda No. 2265 (8769, 9306 and 93414), 1945 (1950), 68 pp.

Aerofoil theory of a flat delta wing at supersonic speeds, Ministry of Supply (London), Aeronaut. Res. Council, Rep. and Memoranda No. 2548 (10,222), 1946 (1952), 21 pp. MR 14 # 699.

The characterization of algebraic plane curves (with Th. Motzkin), Duke Math. J. **14** (1947), 837–853. MR 9 # 373.

Note on the application of the linearised theory for compressible flow to transonic speeds (with A. D. Young), Ministry of Supply (London), Aeronaut. Res. Council, Rep. and Memoranda No. 2399 (10,474), 1947 (1951), 6 pp.

The effect of the sweepback of delta wings on the performance of an aircraft at supersonic speeds (with F. T. Davies), Ministry of Supply (London), Aeronaut. Res. Council, Rep. and Memoranda No. 2476 (10,594), 1947 (1951), 6 pp.

Interference on a wing due to a body at supersonic speeds (with S. Kirkby), Ministry of Supply (London), Aeronaut. Res. Council, Rep. and Memoranda No. 2500 (10,631), 1947 (1952), 10 pp. MR 9 # 479, MR 13 # 882.

Rotary derivatives of a delta wing at supersonic speeds, J. Royal Aeronaut. Soc. **52** (1948), 735–752.

On some problems of unsteady supersonic aerofoil theory, Proc. 7th Internat. Congress Appl. Mech. **2** (1948), 500–514. MR 11 # 477.

Bound and trailing vortices in the linearised theory of supersonic flow, and the downwash in the wake of a delta wing (with J. H. Hunter-Tod), Ministry of Supply (London), Aeronaut. Res. Council, Rep. and Memoranda No. 2409 (11,296), 1947 (1952), 14 pp. MR 9 # 479.

On source and vortex distributions in the linearised theory of steady supersonic flow, Quart. J. Mech. Appl. Math. **1** (1948), 408–432. MR 10 # 410.

The aerodynamic derivatives with respect to sideslip for a delta wing with small dihedral at zero incidence at supersonic speeds (with J. H. Hunter-Tod), Ministry of Supply (London), Aeronaut. Res. Council, Rep. and Memoranda No. 2410 (11,322), 1947 (1952), 14 pp.

On non-associative systems, Proc. Edinburgh Math. Soc. (2) **8** (1949), 111–118. MR 12 # 5.

On the integration of hyperbolic differential equations, J. London Math. Soc. **25** (1950), 209–217. MR 12 # 338.

On functional transformations and summability, Proc. London Math. Soc. (2) **52** 1947 (1950), 132–160. MR 12 # 253.

Les rapports entre le calcul déductif et l'interprétation sémantique d'un système axiomatique, Colloques Internationaux du Centre National de la Recherche Scientifique, Paris, No. 36, 1950, pp. 35–52, Centre National de la Recherche Scientifique, Paris, 1953. MR 15 # 190.

On the application of symbolic logic to algebra, Proc. of the Internat. Congress of Mathematicians, Cambridge, Mass., Vol. 1, 1950, pp. 686–694, Amer. Math. Soc. Providence, R.I., 1952. MR 13 # 716.

Wave reflexion near a wall, Proc. Cambridge Philos. Soc. **47** (1951), 528–544. MR 12 # 875.

On axiomatic systems which possess finite models, "Methodos" Editrice La Fiaccola, Milano (1951), 140–149.

Aerofoil theory for swallow tail wings of small aspect ratio, Aeronaut. Quart. **4** (1952), 69–82. MR 14 # 219, also MR 12 # 452.

L'application de la logique formelle aux mathématiques, in *Applications scientifiques de la logique mathématique*, Actes du 2e. Colloque International de Logique Mathematique, Paris, 1952, pp. 51–64, Gauthier-Villars, Paris, 1954. MR 16 # 782.

Non-uniform supersonic flow, Quart. Appl. Math. **10** (1953), 307–319. MR 14 # 511.

Flow round compound lifting units, Proc. Symposium on High Speed Aerodynamics, Ottawa, Canada, 1953, High Aerodynamics Lab. National Aeronaut. Establishment, Ottawa, 1953, pp. 26–29.

Core-consistency and total inclusion for methods of summability (with G. G. Lorentz), Canad. J. Math. **6** (1954), 27–34. MR 15 # 618.

On some problems of unsteady aerofoil theory, Proc. of the Second Canadian Symposium on Aerodynamics, Toronto, 1954, The Institute of Aerophysics, University of Toronto, 1954, pp. 106–122. MR 17 # 310.

On predicates in algebraically closed fields, J. Symbolic Logic **19** (1954), 103–114. MR 15 # 925.

Note on an embedding theorem for algebraic systems, J. London Math. Soc. **30** (1955), 249–252. MR 17 # 449.

Mixed problems for hyperbolic partial differential equations (with L.L. Campbell), Proc. London Math. Soc. (3) **5** (1955), 129–147. MR 16 # 1116.

Metamathematical considerations on the relative irreducibility of polynomials (with P. C. Gilmore), Canad. J. Math. **7** (1955), 483–489. MR 17 # 226.

On ordered fields and definite functions, Math. Ann. **130** (1955), 257–271. MR 17 # 822.

Further remarks on ordered fields and definite functions, Math. Ann. **130** (1956), 405–409. MR 17 # 118.

A result on consistency and its application to the theory of definition, Nederl. Akad. Wetensch. Proc. Ser. A **59**, and Indag. Math. **18** (1956), 47–58. MR 17 # 1122.

Note on a problem of L. Henkin, J. Symbolic Logic **21** (1956), 33–35. MR 17 # 817.

Ordered structures and related concepts, in *Mathematical Interpretation of Formal Systems*, North-Holland Publ. Co., Amsterdam, 1955, pp. 51–56. MR 17 # 700.

Completeness and persistence in the theory of models, Z. Math. Logik Grundlagen Math. **2** (1956), 15–26. MR 17 # 1173.

Solution of a problem by Erdos-Gillman-Henriksen, Proc. Amer. Math. Soc. **7** (1956), 908–909. MR 18 # 37.

On the motion of small particles in a potential field of flow, Comm. Pure Appl. Math. **9** (1956), 69–84. MR 17 # 1147.

Wave propagation in a heterogeneous elastic medium, J. Math. Phys. **36** (1957), 210–222. MR 19 # 903.

Transient stresses in beams of variable characteristics, Quart. J. Mech. Appl. Math. **10** (1957), 148–159. MR 19 # 106.

Some problems of definability in the lower predicate calculus, Fund. Math. **44** (1957), 309–329. MR 19 # 1032.

Syntactical transforms (with A. H. Lightstone), Trans. Amer. Math. Soc. **86** (1957), 220–245. MR 19 # 934.

On the representation of Herbrand functions in algebraically closed fields (with A. H. Lightstone), J. Symbolic Logic **22** (1957), 187–204. MR 20 # 4555.

Relative model-completeness and the elimination of quantifiers, Dialectica **12** (1958), 394–407. MR 21 # 1265.

Relative model-completeness and the elimination of quantifiers, in Summaries of Talks, Summer Institute for Symbolic Logic, Cornell Univ., 1957, Communications Research Division, Institute for Defense Analyses, Princeton, N.J., 1958 (2nd. ed., 1960), pp. 155–159.

Proving a theorem (as done by man, logician, or machine), in Summaries of Talks, Summer Institute for Symbolic Logic, Cornell Univ. 1957, Communications Research Division, Institute for Defense Analyses, 1958, pp. 350–352.

Applications to field theory, in Summaries of Talks, Summer Institute for Symbolic Logic, Cornell Univ., 1957, Communications Research Division, Institute for Defense Analyses, 1958, pp. 326–331.

Outline of an introduction to mathematical logic. Parts I, II, III, Canad. Math. Bull. **1** (1958), 41–54, 113–136, 193–208; *Part IV*, Canad. Math. Bull. **2** (1959), 33–42. MR 20 # 5123, 20 # 5124; *Parts III and IV*, MR 21 # 6321.

Solution of a problem of Tarski, Fund. Math. **47** (1959), 179–204. MR 22 # 3690.

On the concept of a differentially closed field, Bull. Res. Counc. Israel **8F** (1959), 113–128. MR 23 # 2323.

Obstructions to arithmetical extension and the theorem of Łoś and Suszko, Nederl. Akad. Wetensch. Proc. Ser. A **62** and Indag. Math. **21** (1959), 489–495. MR 22 # 2544.

Algèbre différentielle à valeurs locales, Atti del VI Congresso dell' Unione Matematica Italiana, Napoli 1959, Edizioni Cremonese, Roma, 1959, 1 p.

Local differential algebra, Trans. Amer. Math. Soc. **97** (1960), 427–456. MR 23 # A148.

On the mechanization of the theory of equations. Bull. Res. Coun. Israel **9F** (1960), 47–70. MR 26 # 4910.

Elementary properties of ordered Abelian groups (with E. Zakon), Trans. Amer. Math. Soc. **96** (1960), 222–236. MR 22 # 5673.

Recent developments in model theory, Proc. of the 1960 Internat. Congress for Logic, Methodology and Philosophy of Science, Stanford Univ. Press, Stanford, Calif., 1962, pp. 60–79. MR 29 # 4668.

Model theory and non-standard arithmetic. Infinitistic methods, in Proc. Sympos. Foundations of Math., Warsaw, 1959, Pergamon Press, N.Y., 1961, pp. 265–302. MR 26 # 32.

On the construction of models, in *Essays on the Foundations of Mathematics* (Fraenkel anniversary volume), Magnes Press, Hebrew Univ., Jerusalem, 1961, pp. 207–217. MR 29 # 23.

On the D-calculus for linear differential equations with constant coefficients, Math. Gaz. **45** (1961), 202–206.

Non-standard analysis, Nederl. Akad. Wetensch. Proc. Ser. **A** 64, and Indag. Math. **23** (1961), 432–440. MR 26 # 33.

A note on embedding problems, Fund. Math. **50** (1961/1962), 455–461. MR 25 # 2946.

Airfoil theory, Chapter 72 in *Handbook of Engineering Mechanics* (W. Flügge, ed.), McGraw-Hill, New York, 1962, 24 pp.

Modern mathematics and the secondary schools, Int. Rev. Ed. **8** (1962), 34–40.

A basis for the mechanization of the theory of equations, in *Computer Programming and Formal Systems*, North-Holland Publ. Co., Amsterdam, 1962, pp. 95–99. MR 26 # 4911.

Local partial differential algebra (with S. Halfin), Trans. Amer. Math. Soc. **109** (1963), 165–180. MR 27 # 3634.

On languages which are based on non-standard arithmetic, Nagoya Math. J. **22** (1963), 83–117. MR 27 # 3532.

On generalized limits and linear functionals, Pacific J. Math. **14** (1964), 269–283. MR 29 # 1534.

Random-access stored-program machines, an approach to programming languages (with C. C. Elgot), J. Assoc. Comput. Mach. **11** (1964), 365–399. MR 30 # 4400.

Between logic and mathematics, I.C.S.U. Review 6, North-Holland Publ. Co., 1964, pp. 218–226. Polish translation in Wiadom. Mat. **9** (1966), 89–96.

Formalism 64, in Proc. Internat. Congress for Logic, Methodology and Philos. Sci., Jerusalem, 1964, North-Holland Publ. Co., Amsterdam, 1965, pp. 228–246. MR 35 # 5281.

On the theory of normal families, Acta Philos. Fenn. **18** (1965), 159–184. MR 32 # 5845.

Topics in non-Archimedean mathematics, in *The Theory of Models*, Proc. 1963 Internat. Sympos. Berkeley, Calif., North-Holland Publ. Co., Amsterdam, 1965, pp. 285–298. MR 33 # 5489.

Solution of an invariant subspace problem of K. T. Smith and P. R. Halmos (with A. R. Bernstein), Pacific J. Math. **16** (1966), 421–431. MR 33 # 1724.

A new approach to the theory of algebraic numbers. I. II, Atti Accad. Naz. Lincei Rend. Cl. Sci. Fis. Mat. Natur. (8) **40** (1966), 222–225 (MR 35 # 5305) and 770–774 (MR 35 # 5306).

On some applications of model theory to algebra and analysis, Rend. Mat. e Appl. (Roma) (5) **25** (1966), 562–592. MR 36 # 2489.

Non-standard theory of Dedekind rings, Nederl. Akad. Wetensch. Proc. Ser A **70** and Indag. Math. **29** (1967), 444–452. MR 38 # 4723.

Non-standard arithmetic, Bull. Amer. Math. Soc. **73** (1967), 818–843. MR 36 # 1319.

Multiple control computer models (with C. C. Elgot, J. D. Rutledge), in *Systems and Computer Science* (Proc. Conf. London, Ont., 1965), Univ. Toronto Press, Toronto, Ont., 1967, pp. 60–76. MR 38 # 4073.

The metaphysics of the calculus, in *Problems in the Philosophy of Mathematics*, North-Holland Publ. Co., Amsterdam, 1967, pp. 28–46.

Some thoughts on the history of mathematics, Compositio Math. **20** (1968), 188–193. MR 37 # 21. Also in *Logic and the Foundations of Mathematics*, dedicated to A. Heyting on his 70th birthday, Groningen, Walters-Noordhoff, 1968.

Model Theory, in *Contemporary philosophy*; *A survey*, Vol. I: *Logic and the Foundations of Mathematics*, edited by R. Klibansky, Firenze, La Nuova Italia Editrice, 1968, pp. 61–73.

On flexural wave propagation on nonhomogeneous elastic plates (with A. E. Hurd), SIAM J. Appl. Math. **16** (1968), 1081–1089.

Topics in nonstandard algebraic number theory, in Proc. of Internat. Sympos. on Applications of Model Theory to Algebra, Analysis and Probability, Pasadena, Calif., 1967, Holt, Rinehart and Winston, New York, 1969, pp. 1–17. MR 42 # 3054.

A set-theoretical characterization of enlargements (with E. Zakon), in Proc. of the Internat. Sympos. on Applications of Model Theory to Algebra, Analysis and Probability, Pasadena, Calif. 1967, Holt, Rinehart and Winston, New York, 1969, pp. 109–122. MR 39 # 1319.

Germs, in Proc. of the Internat. Sympos. on Applications of Model Theory to Algebra, Analysis and Probability, Pasadena, Calif., 1967, Holt, Rinehart and Winston, New York, 1969, pp. 138–149. MR 38 # 4723.

Problems and methods of model theory, in *Aspects of Math. Logic*, C.I.M.E. Corsi 3° Ciclo, Varenna, 1968, Edizioni Cremonese, Roma, 1969, pp. 181–266. MR 41 # 5208.

Compactification of groups and rings and nonstandard analysis, J. Symbolic Logic **34** (1969), 576–588. MR 44 # 1765.

Completing theories by forcing (with J. Barwise), Ann. Math. Logic **2** (1970), 119–142. MR 42 # 7494.

From a formalist's point of view, Dialectica **23** (1970), 45–49.

Elementary embeddings of fields of power series, J. Number Theory **2** (1970), 237–247. MR 41 # 3451.

Forcing in model theory, Symposia Mathematica, Vol. 5, Academic Press, New York, 1971, pp. 69–82. MR 43 # 4651.

Infinite forcing in model theory, Proc. 2nd Scandinavian Logic Sympos. in Oslo, 1970, North-Holland Publ. Co., Amsterdam, 1971, pp. 317–340. MR 50 # 9574.

Forcing in model theory, Proc. Internat. Congress of Mathematicians, Nice, 1970, Gauthier-Villars, Paris, 1971, pp. 245–250.

On the notion of algebraic closedness for non-commutative groups and fields, J. Symbolic Logic **36** (1971), 441–444. MR 45 # 43.

Algebraic function fields and nonstandard arithmetic, in *Contributions to Nonstandard Analysis*, North-Holland Publ. Co., Amsterdam, 1972, pp. 1–14.

Inductive theories and their forcing companions (with E. R. Fisher), Israel J. Math. **12** (1972), 95–107. MR 47 # 3163.

The nonstandard $\lambda : \phi_2^4(x)$: *model I. The technique of nonstandard analysis in theoretical physics* (with P. Kelemen), J. Math. Phys. **13** (1972), 1870–1874; *model II. The standard model from a nonstandard point of view*, 1875–1878.

A limit theorem on the cores of large standard exchange economies (with D. J. Brown), Proc. Nat. Acad. Sci. **69** (1972), 1258–1260.

On the real closure of a Hardy field, in *Theory of Sets and Topology* (in honor of Felix Hausdorff), VEB. Deutsch. Verlag Wissensch., Berlin, 1972, pp. 427–433. MR 40 # 4980.

On bounds in the theory of polynomial ideals. Selected questions of algebra and logic (in English) (A collection dedicated to the memory of A. I. Mal'cev) (Russian), Izdat. "Nauka" Sibirsk. Otdel., Novosibirsk, 1973, pp. 245–252. MR 49 # 2364.

Model theory as a framework for algebra, in *Studies in Model Theory*, MAA Studies in Math., Vol. 8, Math. Assoc. Amer., Washington, D.C., 1973, pp. 134–157. MR 49 # 2365.

Function theory on some nonarchimedean fields, in Papers in the Foundations of Mathematics, Slaught Memorial Papers No. 13, Math. Assoc. Amer., 1973, pp. 87–109. MR 48 # 8464.

Ordered differential fields, J. Combinatorial Theory, Ser. A **14** (1973), 324–333. MR 48 # 2124.

Nonstandard points on algebraic curves, J. Number Theory **5** (1973), 301–327.

Metamathematical problems, J. Symbolic Logic **38** (1973), 500–516. MR 49 # 2240.

Standard and nonstandard number systems (The Brouwer Memorial Lecture 1973), Nieuw Arch. Wisk. (3) **21** (1973), 115–133.

Numbers — What are they and what are they good for? Yale Scientific Magazine **47** (1973), 14–16.

Nonstandard arithmetic and generic arithmetic, in Proc. of the Fourth Internat. Congress for Logic, Methodology and Philosophy of Science, Bucharest, 1971, North-Holland Publ. Co., Amsterdam, 1973, pp. 137–154.

A note on topological model theory, Dedicated to Andrzej Mostowski on his 60th Birthday, Fund. Math. **81** (1973/74), 159–171. MR 49 # 7135.

A decision method for elementary algebra and geometry — revisited, Proc. of the Tarski Symposium, Berkeley, Calif., 1971, Proc. Sympos. Pure Math., Vol. 25, pp. 139–152, Amer. Math. Soc., 1974. MR 51 # 2902.

Enlarged Sheaves, in Proc. Victoria Symposium on Nonstandard Analysis, 1972, Lecture Notes in Mathematics, Vol. 369, Springer-Verlag, N.Y., 1974, pp. 249–260.

The cores of large exchange economies (with D. J. Brown), J. Economic Theory **9** (1974), 245–254.

Nonstandard exchange economies (with D. J. Brown), Econometrica **43** (1975), 41–55.

Concerning progress in the philosophy of mathematics, in Proc. Logic Colloquium at Bristol, 1973, North-Holland Publ. Co., Amsterdam, 1975, pp. 41–52.

On the finiteness theorem of Siegel and Mahler concerning Diophantine Equations (with P. Roquette), J. Number Theory **7** (1975), 121–176. MR 51 # 10222.

Algorithms in algebra, in *Model Theory and Algebra*, A Memorial Tribute to Abraham Robinson, Lecture Notes in Mathematics, Vol. 498, Springer-Verlag, N.Y., 1975, pp. 15–40.

On constrained denotation, in *Selected papers of Abraham Robinson*, Yale University Press, New Haven, Connecticut, to appear.